REFERENCE SHEET

Parent Functions and Transformations of Functions

original, $y = f(x)$
transformed graph

GENERAL FORM

$$y = a \cdot f(b(x - h)) + k$$

a — OUTSIDE · Y-VALUES

Vertical stretch / shrink

$|a| > 1$: stretch by a factor of $|a|$

$0 < |a| < 1$: shrink by a factor of $|a|$

$a < 0$: also reflect about the x-axis

b — INSIDE · X-VALUES

Horizontal stretch / shrink

$|b| > 1$: shrink by a factor of $1/|b|$

$0 < |b| < 1$: stretch by a factor of $1/|b|$

$b < 0$: also reflect about the y-axis

h — INSIDE · X-VALUES

Horizontal shift

$f(x - h)$: right h units

$f(x + h)$: left h units

Example: $f(x - 3)$ moves right 3

k — OUTSIDE · Y-VALUES

Vertical shift

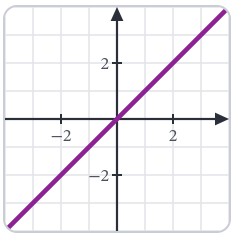
$f(x) + k$: up k units

$f(x) - k$: down k units

Example: $f(x) - 3$ moves down 3

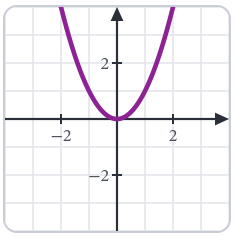
Changes **outside** f affect y-values and do what they look like. Changes **inside** f affect x-values and do the opposite.

PARENT FUNCTION GALLERY



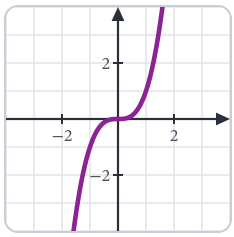
Linear

$f(x) = x$



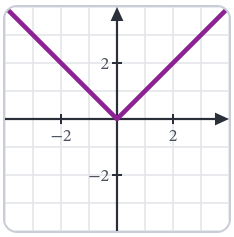
Quadratic

$f(x) = x^2$



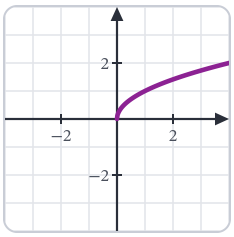
Cubic

$f(x) = x^3$



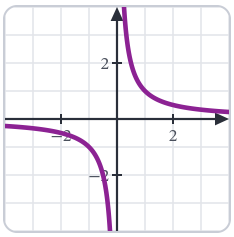
Absolute Value

$f(x) = |x|$



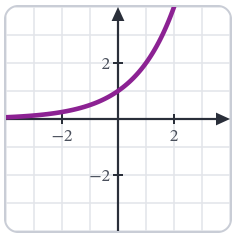
Square Root

$f(x) = \sqrt{x}$



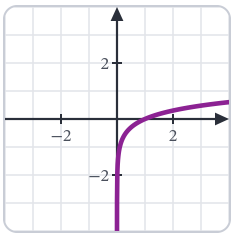
Reciprocal

$f(x) = \frac{1}{x}$



Exponential

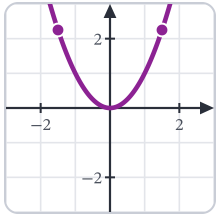
$f(x) = 2^x$



Logarithmic

$f(x) = \log x$

EVEN AND ODD SYMMETRY

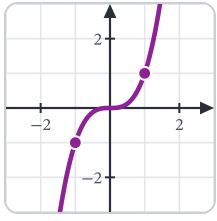


Even

$f(-x) = f(x)$

Symmetric about the y-axis.

Examples: x^2, $|x|$



Odd

$f(-x) = -f(x)$

Symmetric about the origin; the graph looks the same after a 180° turn.

Examples: x, x^3, $1/x$

Neither

Fails both tests.

Examples: $\sqrt{x}$, 2^x, $\log x$

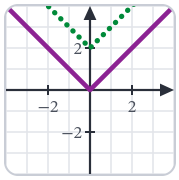
To test: replace x with $-x$, simplify, and compare with $f(x)$ and $-f(x)$.

An even function is unchanged by a reflection about the y-axis. An odd function is unchanged by a reflection about both axes.

Transformations of Functions

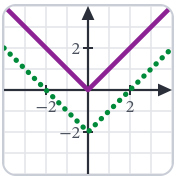
original, $y = f(x)$ — transformed graph

SHIFTS



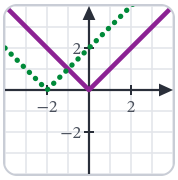
$y = f(x) + 2$

Up 2



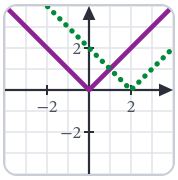
$y = f(x) - 2$

Down 2



$y = f(x + 2)$

Left 2



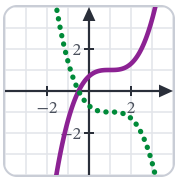
$y = f(x - 2)$

Right 2

Movements up and down change the y-values of points.
Transformations that affect the y-values are intuitive.
For example, to move up 2, you add 2 to the previous y-value.

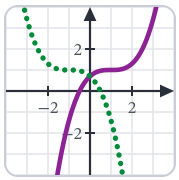
Movements left and right change the x-values of points.
Transformations that affect the x-values are counter-intuitive.
For example, to move left 2, you replace every x by $x + 2$, *not* $x - 2$.

REFLECTIONS



$y = -f(x)$

Reflect about the x-axis



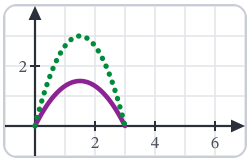
$y = f(-x)$

Reflect about the y-axis

REMEMBER

- **Outside** f changes y-values and does what it says: $f(x) + 2$ moves up 2.
- **Inside** f changes x-values and does the opposite: $f(x + 2)$ moves left 2, and $f(2x)$ shrinks.
- Shifting up/down left/right does *not* change the shape of a graph.
- Vertical/horizontal stretching/shrinking usually changes the shape of a graph.

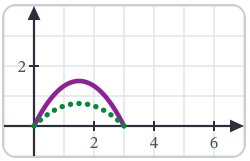
STRETCHING AND SHRINKING



$y = 2f(x)$

Vertical stretch

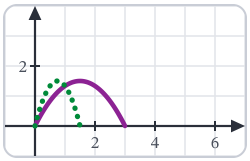
y-values are doubled; points get farther away from x-axis



$y = \frac{f(x)}{2}$

Vertical shrink

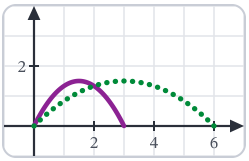
y-values are halved; points get closer to x-axis



$y = f(2x)$

Horizontal shrink

x-values are halved; points get closer to y-axis



$y = f(\frac{x}{2})$

Horizontal stretch

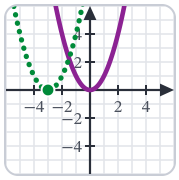
x-values are doubled; points get farther away from y-axis

Vertical stretching/shrinking changes the y-values of points.
Transformations that affect the y-values are intuitive.
$y = a \cdot f(x)$: stretch if $|a| > 1$, shrink if $0 < |a| < 1$, by a factor of $|a|$

Horizontal stretching/shrinking changes the x-values of points.
Transformations that affect the x-values are counter-intuitive.
$y = f(bx)$: shrink if $|b| > 1$, stretch if $0 < |b| < 1$, by a factor of $1/|b|$

WORKED EXAMPLES

EXAMPLE 1 — SINGLE MOVE



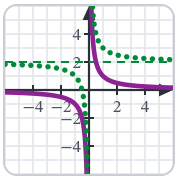
Parent: $y = x^2$

$y = (x + 3)^2$

1. Left 3

Vertex moves from $(0, 0)$ to $(-3, 0)$.

EXAMPLE 2 — SINGLE MOVE



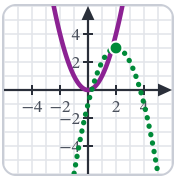
Parent: $y = 1/x$

$y = \frac{1}{x} + 2$

1. Up 2

Horizontal asymptote moves from $y = 0$ to $y = 2$.

EXAMPLE 3 — COMBINED

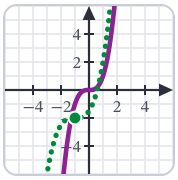


Parent: $y = x^2$

$y = -(x - 2)^2 + 3$

1. Right 2
2. Reflect about the x-axis
3. Up 3

EXAMPLE 4 — COMBINED



Parent: $y = x^3$

$y = \frac{1}{2}(x + 1)^3 - 2$

1. Left 1
2. Vertical shrink by a factor of ½
3. Down 2