Polar Coordinate System

What Are Polar Coordinates?

In contrast to the Cartesian (rectangular) coordinate system, which defines a point by its **horizontal** and **vertical** distances *(x, y)* from a fixed origin, the polar coordinate system expresses a point's location using:

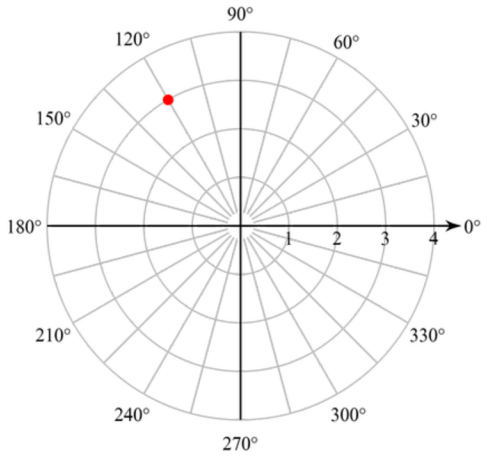
1. r – the radial distance from the origin (how far the point is from the origin).
2. θ – the angle (in degrees or radians) measured counterclockwise from the positive x-axis (how far the point has rotated from the standard direction).

Thus, a point in the plane is represented as *(r, θ)* in **polar coordinates**, where:

- r is always **non-negative** (distance cannot be negative, but direction can be adjusted by modifying θ).
- θ can take values in various conventions: typically $0^\circ \leq \theta < 360^\circ$ (or $0 \leq \theta < 2\pi$ in radians), but negative angles or angles beyond one full rotation are also valid.

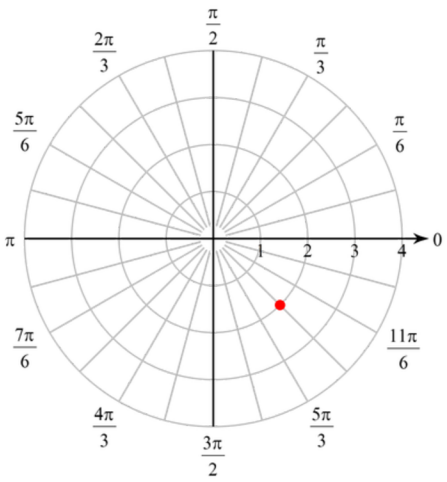
Example 1 (in degrees):

The point $(3, 120^\circ)$ in polar coordinates is 3 units away from the origin and is positioned at a 120-degree angle counterclockwise from the positive x-axis.



Example 2 (in radians):

The point $(2, \frac{7\pi}{4})$ in polar coordinates is 2 units away from the origin and is positioned at a $7\pi/4$ angle counterclockwise from the positive x-axis.



Polar Coordinate System

Conversion Between Polar and Rectangular Coordinates

Since both coordinate systems describe the same space but from different perspectives, conversion formulas allow us to switch between them.

1. Converting from Polar to Rectangular (Cartesian) Coordinates

Given a point *(r, θ)* in polar form, we can find its rectangular coordinates *(x, y)* using the relationships:

Formulas:

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

These formulas arise from interpreting *(x, y)* as the projections of *r* onto the *x*-and *y*-axes in a right triangle.

2. Converting from Rectangular to Polar Coordinates

Given a point *(x, y)* in Cartesian form, we find *r* and *θ* as follows:

Formulas:

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(\tfrac{y}{x})$$

However, care must be taken with *θ*, as $\tan^{-1}(y/x)$ gives an angle in the range $(-90^\circ, 90^\circ)$ (or $-\frac{\pi}{2}, \frac{\pi}{2}$ radians), meaning adjustments are needed depending on which quadrant *(x, y)* falls into.

Quadrant Considerations

If $x > 0, y > 0$ (Quadrant I): $\theta = \tan^{-1}(y/x)$.
If $x < 0, y > 0$ (Quadrant II): $\theta = 180^\circ + \tan^{-1}(y/x)$ (or $\pi + \tan^{-1}(y/x)$ in radians).
If $x < 0, y < 0$ (Quadrant III): $\theta = 180^\circ + \tan^{-1}(y/x)$.
If $x > 0, y < 0$ (Quadrant IV): $\theta = 360^\circ + \tan^{-1}(y/x)$(or $\pi + \tan^{-1}(y/x)$ in radians).

Applications & Significance

- Physics & Engineering: Circular motion, waves, and electromagnetism often benefit from polar representation.
- Navigation & Robotics: GPS systems and robotic pathfinding frequently use polar coordinates.