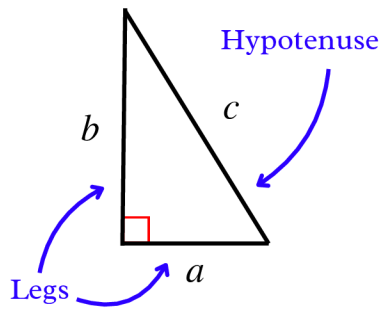
The Pythagorean Theorem

For any right triangle with legs of length a and b, and a hypotenuse of length c, the following relationship holds:

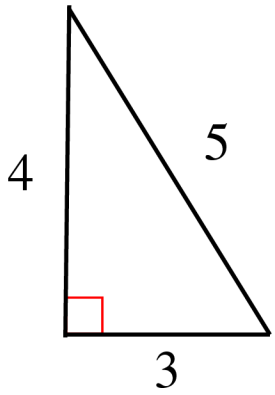
$$a^2 + b^2 = c^2.$$

Definition:



A right triangle with legs of length a and b, and a hypotenuse of length c.

Example:

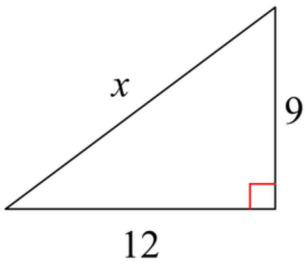


$$a^2 + b^2 = c^2$$

a=3, b=4, c=5

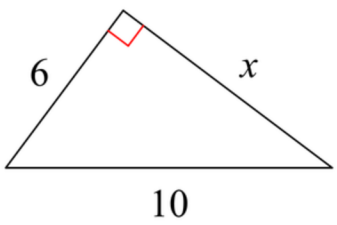
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

Find the length of the hypotenuse, *c:*



$a^2 + b^2$	=	c^2	• The Pythagorean Theorem
$9^2 + 12^2$	=	x^2	• Plug in values: a = 9, b = 12, and c = x
$81 + 144$	=	x^2	• Square 9 and 12 to get 81 and 144
225	=	x^2	• Add 81 and 144 to get 225
$\sqrt{225}$	=	$\sqrt{x^2}$	• Take square root of both sides
15	=	x	• Square root of 225 is 15
x	=	15	• x = 15

Find the length of a leg, *a* or *b:*



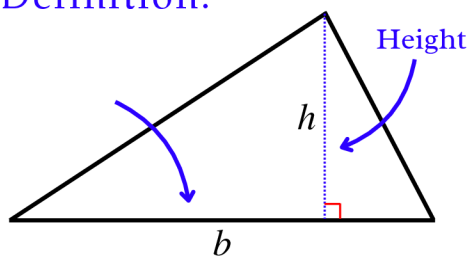
$a^2 + b^2$	=	c^2	• The Pythagorean Theorem
$6^2 + x^2$	=	10^2	• Plug in values: a = 6, b = x, and c = 10
$36 + x^2$	=	100	• Square 6 and 10 to get 36 and 100
x^2	=	$100 - 36$	• Subtract 36 from both sides
x^2	=	64	• 100 minus 36 is 64
$\sqrt{x^2}$	=	$\sqrt{64}$	• Take square root of both sides
x	=	8	• The square root of 64 is 8, so x = 8

Triangle Area

For any triangle with height of length h and a base of length b, the area A is as follow:

$$A_T = \frac{1}{2}bh.$$

Definition:

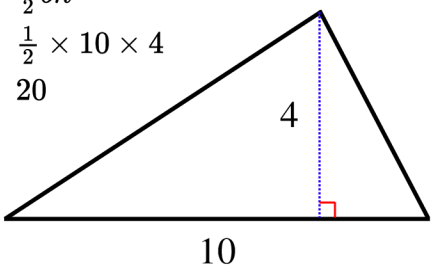


A triangle with a height of length h and a base of length b.

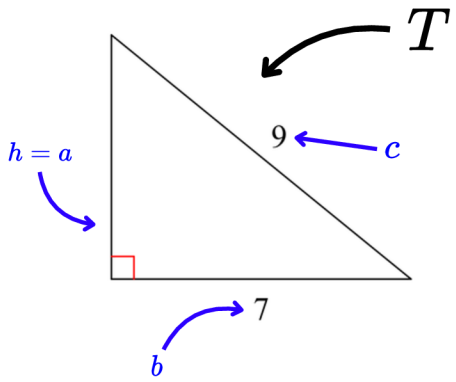
Example:

- Height of length 4
- Base of length 10

$$\begin{aligned} A_T &= \tfrac{1}{2}bh \\ A_T &= \tfrac{1}{2} \times 10 \times 4 \\ A_T &= 20 \end{aligned}$$



Find the area of the triangle.



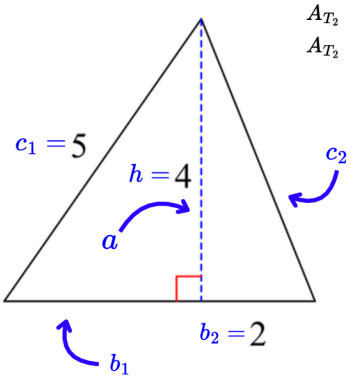
1. Find h

$$\begin{aligned} a^2 + b^2 &= c^2 \\ h^2 + 7^2 &= 9^2 \\ h^2 + 49 &= 81 \\ h^2 &= 81 - 49 \\ h^2 &= 32 \\ \sqrt{h^2} &= \sqrt{32} \\ h &\approx 5.7 \end{aligned}$$

2. Find A_T

$$\begin{aligned} A_T &= \tfrac{1}{2}bh \\ A_T &= \tfrac{1}{2} \times 7 \times 5.7 \\ A_T &= 20 \end{aligned}$$

Find the area of the triangle.



1. Find A_{T_2}

$$\begin{aligned} A_T &= \tfrac{1}{2}bh \\ A_{T_2} &= \tfrac{1}{2}b_2h \\ A_{T_2} &= \tfrac{1}{2} \times 2 \times 4 \\ A_{T_2} &= 4 \end{aligned}$$

2. Find b_1

$$\begin{aligned} a^2 + b^2 &= c^2 \\ h^2 + b_1^2 &= c_1^2 \\ 4^2 + b_1^2 &= 5^2 \\ 16 + b_1^2 &= 25 \\ b_1^2 &= 25 - 16 \\ b_1^2 &= 9 \\ \sqrt{b_1^2} &= \sqrt{9} \\ b_1 &= 3 \end{aligned}$$

3. Find A_{T_1}

$$\begin{aligned} A_T &= \tfrac{1}{2}bh \\ A_{T_1} &= \tfrac{1}{2} \times b_1 \times h \\ A_{T_1} &= \tfrac{1}{2} \times 3 \times 4 \\ A_{T_1} &= 6 \end{aligned}$$

4. Find A_T

$$\begin{aligned} A_T &= A_{T_1} + A_{T_2} \\ A_T &= 4 + 6 \\ A_T &= 10 \end{aligned}$$